Luminous Intensity Distribution Function in Vicinity of Cylindrical and Strip Radiating Lambertian Surfaces

P.D.Rančić (CIE ind. mem., IEEE mem.),
D.D.Vučković, S.Ž.Đokić (IESNA mem.), and D.G.Zulkić

Abstract: General expressions of the luminous intensity distribution function (LIDF) for fluorescent lamp (FL) and light sources in a form of a strip line (SL) with finite length, are presented in this paper. The derived expressions are tested by numerical experiments and illustrated by appropriate diagrams. The results enable accurate calculations of the LIDF for such sources, as well as calculations of total luminous flux of fluorescent lamp with only one illuminance measurement. The mathematical model is adapted to experimental model for LIDF measurement.

1. INTRODUCTION

The bare fluorescent lamps arranged in (continuos) linear rows are not so often, but still are used in the lighting design practice. Also, recessed luminaires with strip shaped opening in the level of the ceiling are used in contemporary interior designs. Sometime, this strip is actually opening in the cylindrical body of the luminaires (luminaires with bar or pole shapes). The LIDF is necessary reference for the precise lighting characteristics calculations in closed spaces illuminated with such light sources.

In previous paper [1], the first author propose general approach for the analytically solution of the LIDF in the near and far field zones, in the case of rectangular light source with ideally diffuse exterior surfaces. In this paper, that general approach is not used only for LIDF calculation in the two common cases (fluorescent lamp and luminous strip line), but is also used for indicating the fact that model enables simple measurement of total luminous flux of the FL (without Ulbricht's sphere). Mathematical model presented in papers [2], [3] and [4] is numerically tested and analytically improved, i.e. limitations of the model validity are introduced, what is not precised in [1]. In this paper, the proposed mathematical model is applied to the cylindrical geometry of the FL, and supported with the number of the numerical experiments.

2. THEORETICAL BACKGROUND

2.1. Light source shaped as thin tube with ideally diffuse radiated surface

Consider thin tube (fluorescent lamp, FL) with length l and diameter d, $d << l$, which exterior surface can be treated as ideally diffuse (Lambertian), with known luminance L. FL could be placed in vertical or horizontal (usual) position, as is shown in Figs. 1a and 1b. In both cases, O_uvw coordinate system is associated to the lamp, with origin in the center of the lamp.

General expression, given in [1], is used for the LIDF calculation,

$$I(\gamma,\varphi) = L\int_{S'}\left(\frac{R}{r}\right)^4\left(\left(\hat{R}-\frac{\vec{r}\,'}{R}\right)\cdot\hat{n}'\right)\left(1-\frac{\vec{r}\,'\cdot\vec{R}}{R^2}\right)\mathrm{d}S'\,, \tag{1}$$

where $\vec{r}\,'$: position vector of the elementary surface dS'. Other quantities are shown in Fig. 1.

From the Fig. 1a, 1b and 1c it is obvious that $\vec{r}\,'\cdot\vec{n}\,'=0$, $\hat{R}\cdot\hat{n}'=non\,f(\vec{r}\,')$ and $\vec{R}=R_u\hat{u}+R_v\hat{v}+R_w\hat{w}=R\sin\gamma\cos\varphi\,\hat{u}+R\sin\gamma\sin\varphi\,\hat{v}+R\cos\gamma\,\hat{w}$, so (1) become simpler in form,

$$I(\gamma,\varphi) = (L\,l\,d)(\hat{R}\cdot\hat{n}')\frac{R^2}{l\,d}\int_{S'}\frac{R^2-\vec{r}\,'\cdot\vec{R}}{r^4}\,\mathrm{d}S'\,, \tag{2}$$

where $S'=ld$: projected radiated surface, which is from the point P viewed as rectangle.

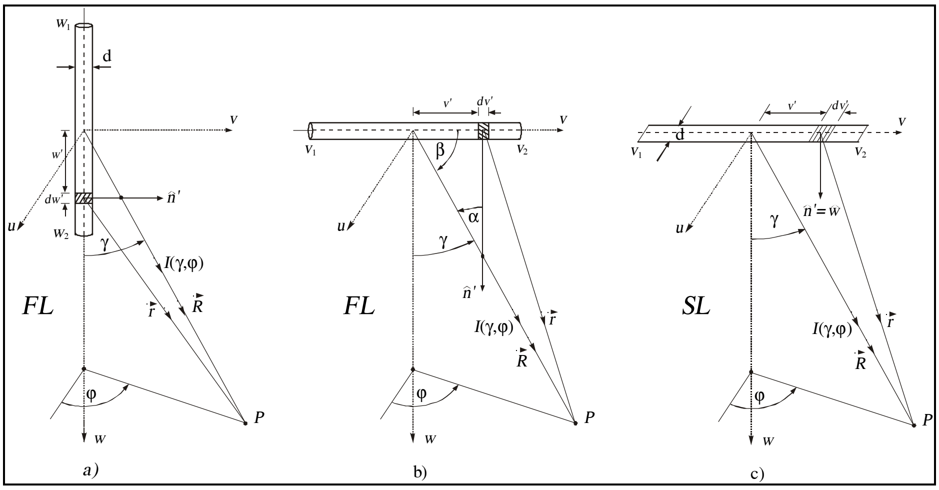


Fig. 1. Illustrations for the LIDF calculation for thine tube placed in vertical and horizontal positions (a and b), and for strip line (c).

Value of Lld represents luminous intensity in $\gamma = 90^o$ direction in Fig. 1a, i.e., according to the Fig. 1b, in $\varphi = 0^o$ direction. In following text this value is denoted as I_0. Expressions (1) and (2) are adopted according to experimental model for measurment of light sources LIDF with sistem photogoniometer.

2.1.1. Lambertian thin tube in vertical position

In this case $\vec{r}' = w'\hat{w}$, $\hat{n}' = \cos\varphi\,\hat{u} + \sin\varphi\,\hat{v}$ and $dS' = d\,\mathrm{d}w'$, what, with substitution in (2), gives

$$I_v(\gamma,\varphi) = I_0 \sin\gamma F_v(\gamma,\varphi) = I_0 \sin\gamma \frac{R^2}{l} \int_{w_1=-l/2}^{w_2=l/2} \frac{R_u^2 + R_v^2 - R_w(w' - R_w)}{\left[R_u^2 + R_v^2 + (w' - R_w)^2\right]^2}\,\mathrm{d}w' \cdot \tag{3}$$

Last integral (function signed as $F_v(\gamma,\varphi)$), is a sum of the two integrals, both with closed form solutions. From (3), general solution for $F_v(\gamma,\varphi)$ is

$$F_v(\gamma,\varphi) = \frac{R^2}{2l} \sum_{i=1}^{2} (-1)^i \left[\frac{w_i}{R_{uv}^2 + (w_i - R_w)^2} + \frac{1}{R_{uv}} \operatorname{arctg} \frac{w_i - R_w}{R_{uv}} \right], \tag{4}$$

where: $w_1 = -l/2$, $w_2 = +l/2$, $R_{uv} = \sqrt{R_u^2 + R_v^2} = R\sin\gamma$ and $R_w = R\cos\gamma$.

Expression (4) is valid for any angle $\varphi \in [0,2\pi], \gamma \in [0,\pi]$ and for $R > l/2$. It is important to emphasize that in the very near field zone (inside the sphere with radius $R = l/2$), expressions (1) - (4) is not valid, i.e. near field zone, according to (1), represents space between sphere with radius $R > l/2$ and approximately $R \approx (5 \div 10)\,l$; outside that space, for $R > (5 \div 10)\,l$ is a far field zone. In this sense, following boundaries can be established:

- The very near field zone, $R < l/2$.
- The near field zone, $l/2 < R < approx.(5 \div 10)\,l$, with two subzones:
  - first one, $l/2 < R < approx.(1.5 \div 3)\,l$, and
  - second one, $approx.(1.5 \div 3)\,l < R < approx.(5 \div 10)\,l$.
- The far field zone, $R > approx.(5 \div 10)\,l$.

Boundary between these three zones can not be defined exactly. However, if maximum requested error through calculation is defined, boundaries are defined, also.

Analysis of general expression (1), i.e. (2), lead to the quite simple form of the LIDF for FL placed in vertical position, at the points in the far field zone, $R >> l$ and $R \approx r$,

$$I_v(\gamma,\varphi) = I_0 \sin\gamma, \tag{5}$$

where: $I_0 = L\ l\ d$ equivalent luminous intensity in the *v-axis* direction. From (4) and (5) follow that for $R >> l$, $F_v(\gamma,\varphi) \to 1$. This state can be founded in related bibliography (for example in [5], [6] and other).

2.1.2 Lambertian thin tube in horizontal position

When FL is placed in horizontal position, then, according to (2) and Fig. 1b, $\vec{r}' = v'\hat{v}$, $\hat{R}\cdot\hat{v} = \sin\gamma\sin\varphi,\ \hat{R}\cdot\hat{n}' = \sqrt{1-\sin^2\gamma\sin^2\varphi}$, what give form similar to (3)

$$I_h(\gamma,\varphi) = I_0 \sin\beta\, F_h(\gamma,\varphi) = I_0 \cdot \sqrt{1-\sin^2\gamma\sin^2\varphi} \cdot \frac{R^2}{l} \int_{v_1=-l/2}^{v_2=l/2} \frac{R_u^2 + R_w^2 - R_v(v'-R_v)}{\left[R_u^2 + R_w^2 + (v'-R_v)^2\right]^2} \mathrm{d}v' . \tag{6}$$

The solution for the function $F_h(\gamma,\varphi)$ is obtained according to (4), by simple change of the *v* and *w* indexes, i.e.

$$F_h(\gamma,\varphi) = \frac{R^2}{2l} \sum_{i=1}^{2} (-1)^i \left[\frac{v_i}{R_{uw}^2 + (v_i - R_v)^2} + \frac{1}{R_{uw}} \operatorname{arctg} \frac{v_i - R_v}{R_{uw}} \right], \tag{7}$$

where $v_1 = -l/2$ and $v_2 = +l/2$.

With above distinction between very near, near and far field zone, can be concluded that at the points in the far field zone, $R >> l$, function F_h, just as function F_v, converges to the one, $F_h(\gamma,\varphi) \to 1$. Then, LIDF get rather simpler form

$$I_h(\gamma,\varphi) = I_0 \cdot \sqrt{1-\sin^2\gamma\sin^2\varphi} . \tag{8}$$

Expressions (6)-(8) can be also obtained by revolving coordinate system from Fig. 1a with (3)-(5).

2.2. Light source shaped as narrow luminous strip line with ideally diffuse surface

If general model from the first part of the Chapter 2.1 and Section 2.1.2 is adapted for the radiated ideally diffuse strip with length l and width d, $d << l$, as is shown in Fig. 1c, then, considering $\hat{n}' = \hat{w}$ and $\vec{r}' = v'\hat{v}$, follow

$$I_t(\gamma,\varphi) = I_0\ \cos\gamma\ F_t(\gamma,\varphi) = I_0\ \cos\gamma\ \frac{R^2}{l} \int_{v_1=-l/2}^{v_2=l/2} \frac{R_u^2 + R_w^2 - R_v(v'-R_v)}{\left[R_u^2 + R_w^2 + (v'-R_v)^2\right]^2} \mathrm{d}v' . \tag{9}$$

The solution for $F_t(\gamma,\varphi)$ is the same as in the case of the horizontally positioned FL, i.e.

$$F_t(\gamma,\varphi) = F_h(\gamma,\varphi) = \frac{R^2}{2l} \sum_{i=1}^{2} (-1)^i \left[\frac{v_i}{R_{uw}^2 + (v_i - R_v)^2} + \frac{1}{R_{uw}} \operatorname{arctg} \frac{v_i - R_v}{R_{uw}} \right]. \tag{10}$$

Again, bearing in mind previous considerations about very near, near and far field zones, LIDF at the points in the far field zone, now with care about scalar product $\hat{R}\cdot\hat{n}'$, is

$$I_t(\gamma,\varphi) = I_0\ \cos\gamma , \tag{11}$$

and total radiated luminous flux of the SL is

$$\Phi_{SL} = 2\pi \int_{\gamma=0}^{\pi/2} I_0 \cos\gamma \sin\gamma \mathrm{d}\gamma = \pi I_0 . \tag{12}$$

Naturally, SL also can be placed in vertical position.

2.3. Measurement of the FL luminous flux

Expressions (6), (7) and (8), as previous, (3), (4) and (5), indicate that presented analysis can be used for simple, mediate measurement of total radiated (installed) luminous flux of the FL. For the such measurement, simple measuring place arrangement and quality luxmeter are required.

Proof: If closed space with ideally absorbing surfaces (for example black plush) is arranged, and if at the one its end FL is mounted, and at the other end, at distance D, D in [m], illuminance meter is placed, then, from Fig. 2a

$$E_{mer} = \frac{I_v(\gamma,\varphi)}{D^2} = \frac{I_v(\gamma=90,\varphi=0)}{D^2} = \frac{I_0}{D^2}\cdot F_v(\gamma=90,\varphi=0)\,, \tag{13a}$$

and from Fig. 2b

$$E_{mer} = \frac{I_h(\gamma,\varphi)}{D^2} = \frac{I_h(\gamma=0,\varphi=0)}{D^2} = \frac{I_0}{D^2}\cdot F_h(\gamma=0,\varphi=0)\,, \tag{13b}$$

i.e. luminous intensity can be calculated as $I_0 = D^2\,E_{mer} / F_{v/h}$.

From the luminous flux conservation law, radiated (installed) luminous flux of the FL, according to (5) and the presented symmetry about axis of rotation, is

$$\Phi_{FL} = 2\pi\int_{\gamma=0}^{\pi} I_0 \sin\gamma \sin\gamma \mathrm{d}\gamma = 2\pi I_0 \int_{\gamma=0}^{\pi} \frac{1-\cos 2\gamma}{2}\mathrm{d}\gamma = \pi^2 I_0\,, \tag{14a}$$

or alternatively by (8), with some complicated integration, what also lead to the same solution, i.e.

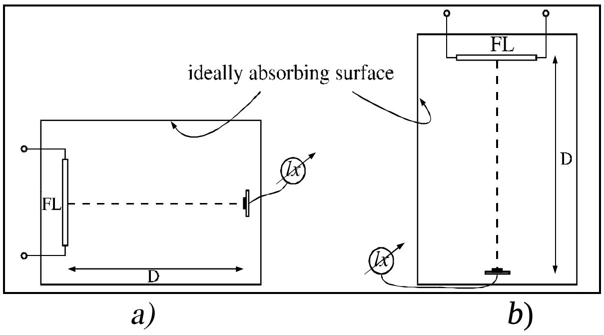


Fig. 2: Measuring conditions for simple measurement of installed luminous flux of the FL.

$$\Phi_{FL} = \int_{\varphi=0}^{2\pi}\int_{\gamma=0}^{2\pi} I_0 \sin\beta \sin\gamma \mathrm{d}\gamma \mathrm{d}\varphi = \pi^2 I_0\,. \tag{14b}$$

By using measured values for I_0 from (13a) in expression (14a), luminous flux of the fluorescent lamp from Fig. 2a is

$\Phi_{FLmeas} = \pi^2 D^2 E_{mer} / F_v(\gamma=90^o,\varphi=0^o)$,

and according to the Fig. 2b,

$\Phi_{FLmeas} = \pi^2 D^2 E_{mer} / F_h(\gamma=0^o,\varphi=0^o)$.

3. NUMERICAL RESULTS

Presented numerical results, obtained through the number of numerical experiments, should illustrate validity of the proposed method and its expressions. In the Figs. 3, 4 and 5 normalized LIDFs for the three considered cases are shown, in dependence of the normalized distance p, $p = 2R/l$, while angles (γ, φ) which define particular directions, are parameters.

Results that illustrate luminous flux conservation law, for all three cases, are organized in the Table 1, where results in third and fourth column are related to the FL (vertical and horizontal position), and those in fifth column are related to the SL (Fig. 1c). All results are given as a function of the normalized distance p. Values calculated by (14a), (14b) and (12), fill the last row.

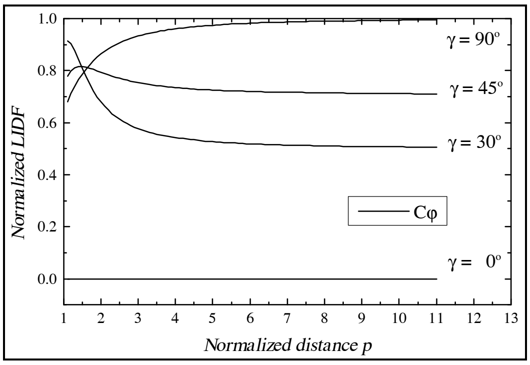
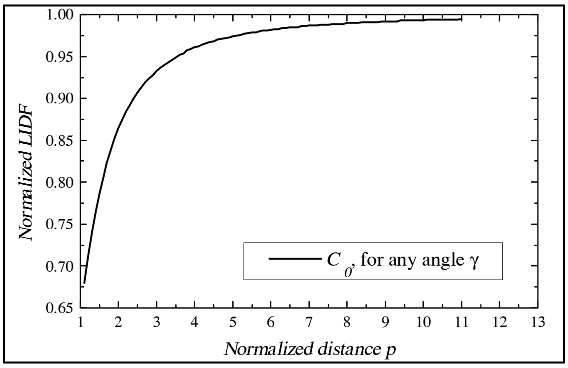
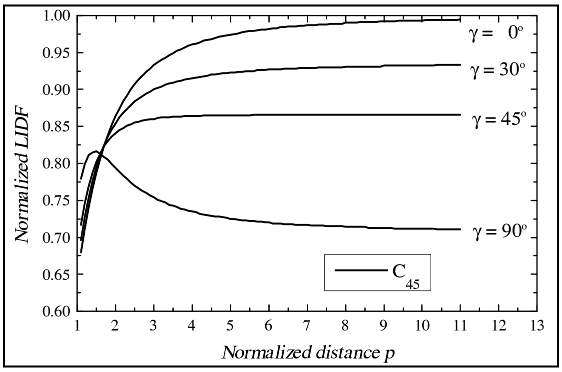


Fig. 3: Normalized LIDFs of the vertically positioned FL versus normalized distance p for any angle φ and with angle γ as parameter.

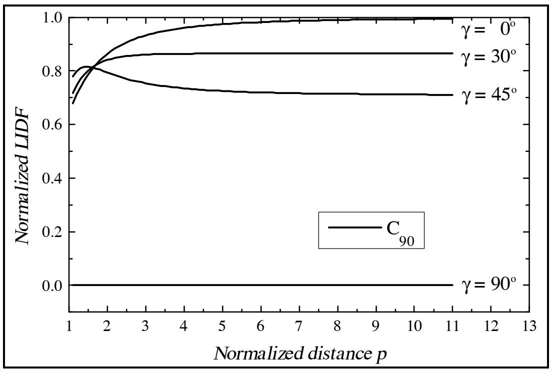
Polar diagrams in the C_φ planes for considered light sources and for different values of normalized distance p as parameter, are drawn in Figs. 6, 7 and 8.



4a



4b



4c

Fig. 4: Normalized LIDFs of the horizontally positioned FL as a function of normalized distance *p* with angle γ as parameter.

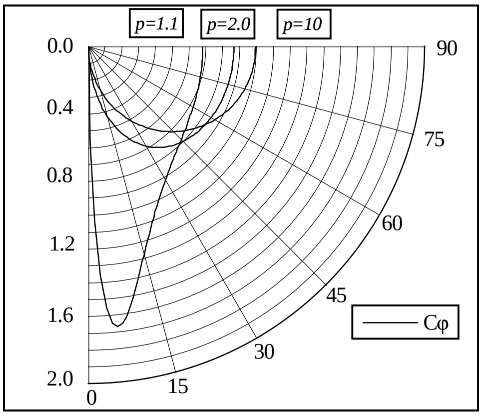
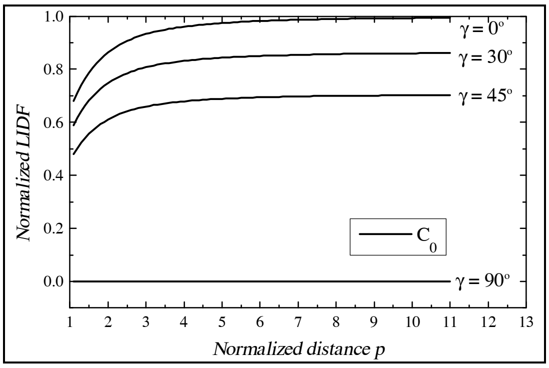
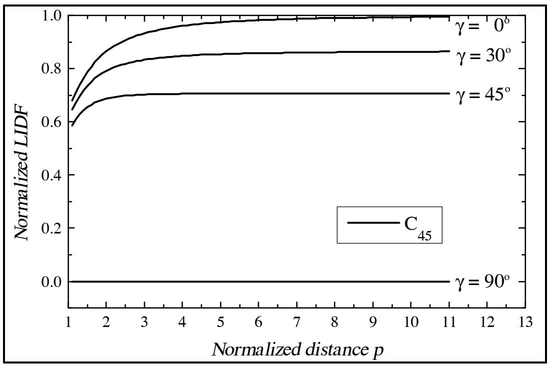


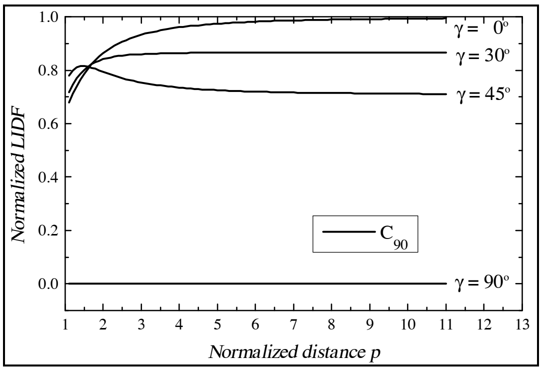
Fig. 6: Polar diagrams of the normalized rotationally symmetrical LIDFs in the case of the vertically positioned FL. *p* is a parameter.



5a



5b



5c

Fig. 5: Normalized LIDFs of the horizontally positioned luminous SL as a function of normalized distance *p* with angle γ as parameter.

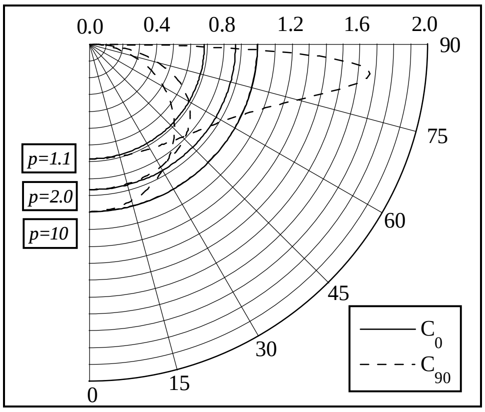


Fig. 7: Polar diagrams of the normalized LIDFs in the case of the horizontally positioned FL given in the C_0 and C_{90} planes. *p* is a parameter.

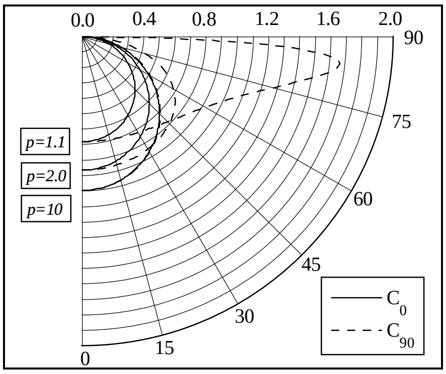


Fig. 8: Polar diagrams of the normalized LIDFs in the case of the luminous SL given in the C_0 and C_{90} planes. *p* is a parameter.

Table 1: Normalized values of the total radiated luminous flux, as a function of the parameter *p*.

No.	Normalized distance *p*	Φ_{FLv}/I_0 [lm/cd]	Φ_{FLh}/I_0 [lm/cd]	Φ_{SL}/I_0 [lm/cd]
1.	1.1	9.8555	9.7677	3.2334
2.	1.5	9.8574	9.7972	3.1432
3.	2.0	9.8590	9.8327	3.1420
4.	2.5	9.8596	9.8424	3.1419
5.	5	9.8598	9.8480	3.1417
6.	10	9.8598	9.8583	3.1416
7.	∞	$\pi^2 = 9.8696$	$\pi^2 = 9.8696$	$\pi = 3.1416$

3. CONCLUSION

The theoretical model for the LIDF calculation of the light sources shaped as FL and SL, is given in this paper. Expressions can be used for various numerical calculations related to the mathematical modeling of the illuminated closed spaces light characteristics.

With performing only one measurement of the luminous intensity, or horizontal illuminance, model also enables precise calculation of the LIDF in the second part of the near zone, as well as in the far field zone. In the first part of the near field zone, LIDF gives only information about directivity of the radiation in radial directions. In the second part of the near field zone and in the far field zone, the LIDF defines directive characteristics and enables substitution with equivalent point light source.In the very near field zone, defined by sphere $p < 1$, presented model can not be used for the LIDF calculation.

Finally, presented model provide possibility for the mediate determinatio-n of the total luminous flux, by the simple, single illuminance measurement.

ACKNOWLEDGMENT: This paper was partially supported by the Ministry of Science and Technology of the Republic of Serbia.

4. REFERENCES

[1]RANČIĆ P D: *Supplements to the Lighting Engineering Characterizations, Volume 4: Lighting of Closed Spaces*, University of Nis, Faculty of Electronic Eng., Niš, 1997. (in Serbian)

[2]RANČIĆ P D, VUČKOVIĆ D D, and ĐOKIĆ S Ž: Contribution to Luminous Intensity Distribution Function Modelling”, Proc. of *CIE LED Symposium 97*, p.p. 31-36, Publ. CIE x013-1997, Vienna, 1997.

[3]RANČIĆ P D, ĐOKIĆ S Ž, and VUČKOVIĆ D D: Theoretical Photometric Characterization of One Type of Luminaires With Opal Diffuser, *J. of Electrotechnics and Mathematics*, No.1, pp. 37-51, Faculty of Electrical Eng. Priština, 1997.

[4]RANČIĆ P D, VUČKOVIĆ D D, and ĐOKIĆ S Ž: “Contribution to Simple Mathematical Characterizations of the Spherical (Scalar) and Hemispherical Illuminance Distributions”, *24th CIE Sesion’99*, Warsaw, Poland, 24-26 June, 1999. (Accepted for poster presentation)

[5]MESHKOV V V: *Fundametals of Illumination Engineering*, Eng. Trans., Mir Publishers Moscow, 1981.

[6]KUSCH O: *Computer-Aided Optical Design of Illuminating and Irradiating Devices*, “ASLAN” Publ. House, Moscow, 1993.

Addresses of the authors:
Prof. dr Predrag D. Rančić
Dipl. ing. Dragan D. Vučković
Dipl. ing. Saša Ž. Đokić
Dipl. ing. Dijana G. Zulkić

Faculty of Electronic Eng.
P.O.Box 73, 18000 Niš, Srbija
Tel: +381 18 52 93 05
Fax: +381 18 46 180
E-mail: prancic@elfak.ni.ac.yu